

GaitMesh: controller-aware navigation meshes for long-range legged locomotion planning in multi-layered environments

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Abstract—Long-range locomotion planning is an important problem for the deployment of legged robots to real scenarios. Current methods used for legged locomotion planning often do not exploit the flexibility of legged robots, and do not scale well with environment size. In this paper we propose the use of navigation meshes for deployment in large-scale multi-floor sites. We leverage this representation to improve long-term locomotion plans in terms of success rates, path costs and reasoning about which gait-controller to use when. We show that NavMeshes have higher planning success rates than sampling-based planners, but are 400x faster to construct and at least 100x faster to plan with. The performance gap further increases when considering multi-floor environments. We present both a procedure for building controller-aware NavMeshes and a full navigation system that adapts to changes to the environment. We demonstrate the capabilities of the system in simulation experiments and in field trials at a real-world oil rig facility.

Index Terms—Legged Robots; Autonomous Vehicle Navigation; Motion and Path Planning.

I. INTRODUCTION

LEGGED robots can use a variety of locomotion gaits. For example, quadruped robots can switch their locomotion mode to walk, trot or bound according to the terrain at hand. This diversity of ways to navigate environments renders legged robots unmatched for overcoming varied terrain. However, it comes at the cost of reasoning about terrain features and choosing the appropriate gait to use for each particular area. Even though the choice of gait-controllers has been included in recent locomotion planning methods [1], it is still not clear which map representations and planning methods are most suitable for long-range locomotion tasks.

In this paper we present our approach to long-range legged robot locomotion planning, building on navigation meshes (NavMeshes) [2], [3], [4]. Our approach was developed having in mind applications of inspection and monitoring of large industrial facilities, e.g. power plants, offshore wind and oil & gas platforms, or nuclear facilities. In this context we assume an approximate map of the environment to be known in advance,

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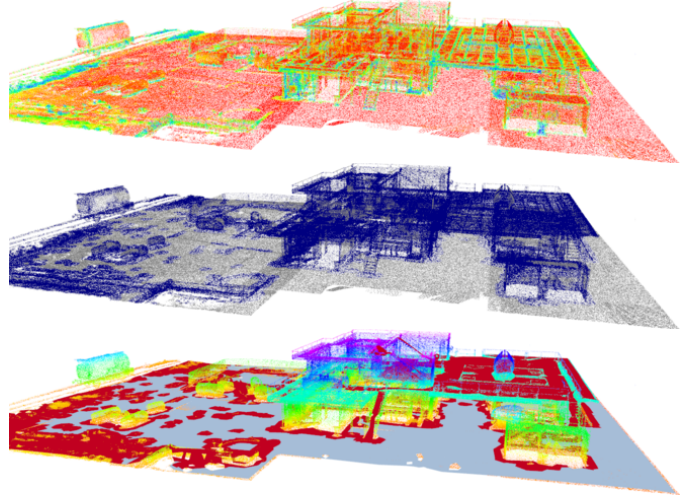


Fig. 1: Top to bottom: point cloud of an oil rig with color-coded curvature, gait-controller choice (gray for trotting and dark blue for walking), navigation mesh (light blue for trotting and red for walking).

or that the robot can be teleoperated to build up a map of the facility before autonomous deployment.

NavMeshes are popular map representations within computer game AI [5], where they are used to plan paths for agents over large-scale environments such as buildings or open-worlds. Such large-scale planning methods are important for autonomous long-term robot deployment. Reasoning about the choice of gait is another requirement for large-scale planning in the context of legged locomotion. For example, going through a building following a small-distance path might involve the use of slow walking gaits to climb stairs and navigate narrow corridors. In contrast, going around the same building on a longer-distance route could be a faster alternative using a speedy trotting gait.

In this paper we present methods for both building and using gait-aware navigation meshes for legged locomotion at such scales. Our contributions are:

- A complete system to navigate large-scale sites while reasoning (and switching between) gait-controllers, using NavMeshes together with local planners.
- An automated method to generate navigation meshes of large-scale environments for legged robots with multiple gait-controllers.
- An evaluation of the advantages of navigation meshes for

long-range planning.

- A detailed comparison between traditional robotics methods such as PRMs and RRTs, and NavMeshes, demonstrating how NavMeshes overall outperform these methods, especially when planning over multiple floors.

The novelty of our system is that it can solve long-range multi-controller locomotion paths, that take gait capabilities of legged robots into account. Compared to previous state-of-the-art legged systems, ours automatically plans long-range paths over multiple floors, and considers gait capabilities for long-range and not just short-range locomotion [1]. The system can also handle obstacle and terrain changes to the environment through the effective use of virtual obstacles in navigation meshes. We evaluate our system both in simulation experiments and on multiple real-robot field trials at a realistic oil rig facility, used for personnel training exercises.

II. RELATED WORK

A large number of map representations have been used for robot locomotion planning. Probabilistic Roadmaps [6], occupancy grids [7], OctoMaps [8] and heightmaps [9], are a few of the popular map representations for this purpose. These are also popular for legged robot locomotion planning. For example, [10] uses search on occupancy grids and [11], [1] on heightmaps, while [12] uses RRT* on heightmaps.

While these representations are suited to short-term planning, they do not scale well to large (e.g. multi-floor) environments. For this reason, in deployments to large environments researchers have used manually designed topological graphs [13], [14]. Related map representations for long-term robot deployments include architectural floor plans [15], hand-drawn floor plans [16], and 3D vector maps [14]. Also for long-term planning, Probabilistic Roadmaps (PRMs) [6] have been used for legged robot locomotion planning [17] and in combination with Reinforcement Learning methods for long-term mobile robot navigation [18]. Outside of robotics, [19] uses a database of CAD models for each floor and building of a university for navigation. While this representation allows long-range planning, it comes at the cost of intense manual labour in terms of CAD drawing, stair labeling, and computing distances between floors and buildings.

Our work targets long-term and large-scale robot deployments, for quadruped robots in particular. We specifically focus on building a system for fast path-finding in large-scale, potentially multi-level environments such as industrial oil rigs and other hard-to-access sites [1]. Except for PRMs, the map representations we have mentioned are either single-floor or manually designed—which could lead to imprecise maps or large manual work requirements. To account for multi-level large-scale facilities while avoiding manual labeling we use point cloud acquisition and automatic navigation-mesh construction from point clouds. Navigation meshes [2], [20], [3], [4] are related to simpler 2D cell-decomposition methods in robot path planning, and also to semantic segmentation methods such as the watershed algorithm [21] and others in the way they partition the space into walkable non-overlapping regions. These map representations are popular in computer games due

to their scalability and fast computation time for path-finding. Computation time is critical in game AI since often paths have to be found for hundreds or thousands of agents at fast rates [22]. The Recast toolkit for navigation meshes [20], for example, is used in several commercial games [23] and in the Unity engine [5]. Navigation meshes represent the world as a set of polygons and a graph representing traversability between them. Path-finding consists in a search over this compact graph. A comparison of navigation mesh methods is made in [4].

Due to requirements of computer games, navigation meshes also usually encode per-polygon labels for the possible modes of locomotion and the costs of different map areas (e.g. an agent could swim over a river or travel to the closest bridge). This functionality is also close to the reality of legged robot locomotion—characterized by a large set of possible gaits [24] and controllers specialized for different kinds of terrain [1]. In this paper we use navigation meshes where each polygon is assigned with a choice of controller. This allows to make long-term plans aware of the real cost of traversing different regions, and hence obtain paths of low global cost. We automatically compute these controller annotations by local 3D point cloud features similar to the heightmap-based work in [12].

III. DEFINITIONS

1) *Gait controller*: In this paper a “gait controller” is any method that controls the full-body motion of a robot to achieve a desired velocity, or goal position, of a robot’s base. We consider a setting where multiple controllers are available for a given robot, specialized to different kinds of terrain. We represent this set of controllers by $\mathcal{M} = m_1, \dots, m_M$, where M is the number of available controllers.

2) *Walkable environment*: We use the concept of a “walkable environment” W as defined in the navigation-mesh literature [4]. W is a set of triangles that are traversable by an agent, i.e. robot. For the purpose of this paper, each triangle t is a tuple of three points and a label, $t = (p_1, p_2, p_3, m)$, where $p_1, p_2, p_3 \in \mathbb{R}^3$ and $m \in \mathcal{M}$. The label identifies the preferred choice of controller to be used when the robot’s projected COM lies within the triangle. These triangles represent the surfaces traversable by the agent (i.e. robot). For typical legged robots this will consist of triangles on the floor, stairs and other traversable surfaces. Surfaces that are more inclined than what is possible by the robot’s capabilities will not be part of the walkable environment.

3) *Multi-layered environment*: Intuitively, a “multi-layered environment” is an environment with walkable areas at multiple heights, e.g. a multi-floor building. More formally, a walkable environment W is multi-layered when the projection of its triangles to a horizontal plane leads to intersections [4].

4) *Navigation mesh*: A navigation mesh is a tuple $N = (W, G)$: it consists of a walkable environment W and an undirected graph G representing the possibility to navigate between adjacent triangles in W .

IV. METHOD

The overview of our proposed system is shown in Fig. 2. It uses high-level planning based on NavMeshes together

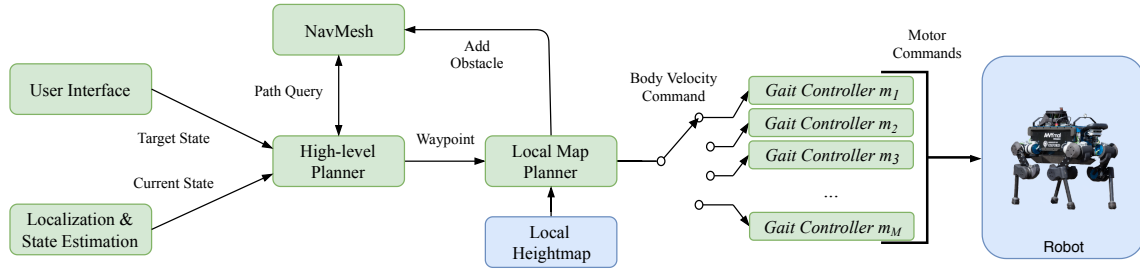


Fig. 2: System overview.

with local-map planning to navigate large environments, while allowing for online adaptation using virtual obstacles. We will go through the process of building NavMeshes, and the content of each of the blocks in the following sections.

A. Building large controller-annotated environment meshes

In order to build a gait-aware NavMesh we require a triangular mesh where each triangle is annotated with a gait controller choice. We will now explain the process, summarized in Fig. 3, to obtain such a mesh. The method uses either a CAD model of the environment or a point-cloud acquired on-site. We start by describing the point-cloud-based method.

1) *Point cloud acquisition:* We obtain point clouds of whole multi-layered facilities by successive laser-scanning and registration using a portable device. In principle any mapping device can be used, including the on-board robot SLAM system, as long as the cloud captures the walkable environment, i.e. floor, staircase steps and other surfaces.

2) *Per-point controller assignment:* We start by computing normals and curvature for all points in the point cloud. After, in a similar way to [12], for each point we compute the local maximum of curvature c_{\max} in a spherical neighborhood around the point. We use the radius of the smallest sphere which encloses the robot for this. While we found that curvature information is sufficient to choose between our robot platform's controllers, other features such as roughness, slope or height-differences could be used [12] as appropriate for the robotic platform and controller choices available.

In our case we assign a trotting-gait controller, specialized for flat terrain, to all points where c_{\max} is below a threshold, and otherwise assign a walking-gait controller which uses vision for foot placement. The output of this step is therefore a point-cloud annotated with a choice of controller (i.e. a unique number identifying a controller).

3) *Mesh reconstruction:* We reconstruct a 3D triangular mesh from the original point cloud using the Ball-Pivoting algorithm [25]. While other methods are openly available [26] we found that Ball-Pivoting produced good results for large-scale environments. We recommend the interested reader to see [27] for a comparison of this and alternative open-source mesh reconstruction methods for robotics applications.

4) *Per-polygon controller assignment:* The final step in the procedure is to assign a controller choice to each *triangle* of the reconstructed mesh. To achieve this we project the point-cloud controller-annotations back to mesh triangles. For each triangle t in the mesh we obtain all the points in the annotated

point cloud P that fall within a distance d_{\max} of t . We then run a voting procedure: we count the number of points which support each of the controllers and assign the maximally-voted controller to triangle t .

When a CAD model of the environment is available, we uniformly sample the CAD model to obtain a dense point cloud. We then run the same cloud-annotation and cloud-to-mesh annotation projection procedure, described above, to obtain a controller-annotated mesh.

B. Building multi-controller navigation meshes

We use Recast [20] to generate navigation meshes from the annotated mesh. Recast starts from a triangular mesh as input, and produces a navigation mesh using the following procedure:

- 1) Voxelize the polygons. This generates a multiple-height heightfield from the triangular mesh, where each cell contains a list of the occupied heights.
- 2) Annotate walkable space from solid voxels. This filters out voxels where a cylindrical approximation of the robot cannot stand based on surface inclination and vertical clearance. The heightfield representation obtained by the previous step is particularly suited for this stage since identifying voxels on floor surfaces and with enough ceiling-clearance is fast.
- 3) Compute a distance field in voxel space. This is the distance of each walkable voxel to the closest non-walkable voxel on the heightfield, or to a voxel annotated with a different controller-choice.
- 4) Run a “watershed” algorithm on the distance field. This basically segments the walkable voxels into simple regions that do not overlap. We encourage the interested reader to refer to [21] for a detailed and intuitive explanation of the method.
- 5) Compute polygons representing the contours of the segmented voxel regions, using the Ramer-Douglas-Peucker algorithm [28].
- 6) Triangulate the polygons and build the connectivity graph G used for planning in the navigation mesh.

For a more in-depth overview of the design choices and implementation, we refer the reader to [2].

When applied to large-scale point-cloud-estimated meshes, the previous procedure generates a considerable number of unconnected (inaccessible) walkable areas—such as the top of a lamp post, the top of ceiling rails, or areas where point cloud noise is high (e.g. far away from the laser scans). This

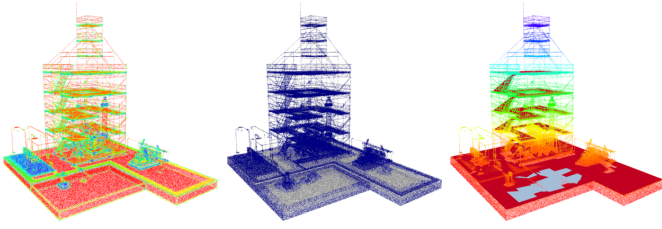


Fig. 4: Environment 2, the many-floored ARGOS Challenge facility model. Left to right, point cloud with color-coded curvature, controller choice, navigation mesh.

walking areas are in red. Floors are connected by stairs which appear connected in the navigation mesh. The number of polygons to represent the full environment is 6075. Most flat ground is annotated with trotting, except around narrow passages such as doorways and staircases. The elevated floor of the facility is annotated with static walking because of its rough grid-like flooring.

Environment 2 is a CAD model of a real oil & gas onshore site used for TOTAL's ARGOS Challenge. We followed the CAD model procedure indicated in Section IV-A. NavMesh generation took 0.5 seconds. We show the clouds and NavMesh in Fig. 4. Multiple floors are connected by stairs which appear as long triangles in the navigation mesh. The number of polygons to represent the environment is 447. Elevated floors are annotated with static-walking as the corridors are narrow.

B. NavMeshes vs. sampling-based roadmaps

In this section we compare NavMeshes against popular sampling-based planning methods in robotics. PRM*, in particular, uses a similar approach to planning compared to NavMeshes—it encodes traversability between locations in the environment offline as a graph, which is then searched at query time and refined to find an optimal path. While NavMeshes rely on deterministically building a geometric representation of the environment (set of triangles), PRMs build only a graph connecting specific random locations. For this experiment, all methods minimized only distance of the path—thus ignoring gait-controller annotations.

We ran the comparison on Environment 1. We used the implementation of PRM* in OMPL [29]. We defined state validity as the intersection of a cylinder with any points on the original cloud. We used a cylinder of the same dimensions as that used for the NavMesh. For a state to be feasible it also needs to intersect points in a volume below the cylinder (i.e. to make contact with the ground). States are sampled by randomly picking points from the cloud and randomly picking a robot height within an interval. For planning trajectories in this roadmap we let PRM* refine the path. We obtained results for building times of 10, 360 and 3600s, and for planning times of 0.1s and 10s. We also compare NavMeshes to the use of RRTs: where at planning time an RRT is run from scratch.

Table I shows the pre-computation time, planning time, and path length obtained with NavMeshes, PRMs and RRTs on 200 planning problems. Each planning problem is a randomly sampled start and goal state, and the average path

lengths are shown. All methods were given the same planning problems. Since randomly-generated start and goal states can be on different-floors, we specifically generate 100 same-floor problems and 100 different-floor problems. We ran the benchmark single-threaded on a Intel Xeon 3Ghz CPU. The table shows that NavMeshes obtain 100% success rate at 100x faster computation times than the best-performing sampling-based method. The best competitor was PRM* when it was left to build the roadmap for 3600s (compared to 8.6s building time for NavMeshes) and at a computation time of 0.1 second, compared to 1ms for NavMeshes. Path lengths were equivalent on average in PRMs, but only when the roadmap is built for long enough time (3600s). In this case, PRMs sometimes obtain shorter paths than the NavMesh's as seen by the standard deviation, which is due to its continuous nature and capability to “cut corners” and leave the NavMesh's polygons close enough to obstacles. They similarly often obtain longer paths due to their stochastic nature, and obtain slightly lower success rates. Of the single query methods RRTConnect did best but only solved 88 out 100 problems (at 10s computation).

Importantly, the success rate of sampling-based planners considerably drops when the environment becomes more challenging and involves narrow passages, such as staircases to access different floors. In this setting, only PRM* manages to find paths, but again at lower success rates and much larger computation times than NavMeshes.

C. Qualitative inspection of multi-controller paths

Next, we qualitatively show the behavior of the NavMesh-based planner when gait-controller annotations are considered. We have two controllers available: 1) flat-ground trotting [30] and 2) vision-based planned-step static walking [31]. Since the trotting controller moves at approximately 8 times faster speeds than the static walking controller, we used a cost multiplier of 8 for walking-annotated regions. This means that the cost of 1 meter on a trotting-annotated region is 1, but on a walking-annotated region it is 8.

Fig. 5 shows paths around a 20cm-high barrier on the ground. It shows how the planner produces paths that go around the barrier when the robot is sufficiently close to its end, but over the barrier when far away. Fig. 6 shows a long-range example, where the path goes around a building for a longer distance instead of going inside and outside the building in a shorter-distance straight path. Going through the building involves slow walking over the stairs to go in and out, when compared to the time required to go around the structure on a flat area while trotting. In this case the predicted travel time would be 3.7 higher going through the building instead of around it, despite the shorter distance.

D. Full execution in simulation

In the next experiment we tested the execution of the full system of Fig. 2 which includes high-level planning, local-map planning and controller switching. We conducted the experiment in the Gazebo simulator in ROS. The robot started in the middle of the (approximate simulation model of the) FSC oil rig facility and we gave the high-level planner a goal inside

TABLE I: Benchmark of success rates and path lengths in NavMeshes vs PRMs and RRTs.

Planner	Pre-computation time (s)	Planning time (ms)	Same-floor problems		Different-floor problems	
			Success rate	Path length $\frac{l_{\text{Planner}}}{l_{\text{NavMesh}}}$	Success rate	Path length $\frac{l_{\text{Planner}}}{l_{\text{NavMesh}}}$
NavMesh	8.6	0.858 ± 0.791	100 / 100	1.00 ± 0.00	100 / 100	1.00 ± 0.00
PRM*	10.0	100	82 / 100	1.13 ± 0.31	00 / 100	-
PRM*	10.0	10000	83 / 100	1.13 ± 0.30	00 / 100	-
PRM*	360.0	100	91 / 100	1.02 ± 0.03	40 / 100	1.29 ± 0.36
PRM*	360.0	10000	91 / 100	1.01 ± 0.03	40 / 100	1.29 ± 0.36
PRM*	3600.0	10	03 / 100	1.00 ± 0.01	08 / 100	0.91 ± 0.17
PRM*	3600.0	100	97 / 100	1.00 ± 0.07	96 / 100	1.02 ± 0.22
PRM*	3600.0	10000	97 / 100	1.00 ± 0.07	96 / 100	1.02 ± 0.22
RRTConnect	0.0	100	09 / 100	1.59 ± 0.32	00 / 100	-
RRTConnect	0.0	10000	88 / 100	2.08 ± 0.88	00 / 100	-
RRT*	0.0	100	04 / 100	1.44 ± 0.27	00 / 100	-
RRT*	0.0	10000	10 / 100	1.16 ± 0.19	00 / 100	-

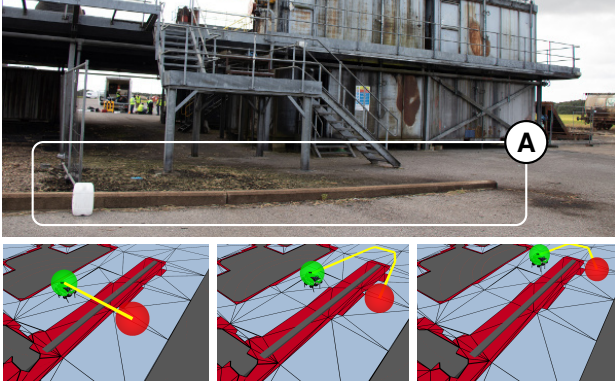


Fig. 5: The influence of controller-reasoning in the NavMesh. The robot either climbs over or trots around a barrier (A) depending on the distance to its end.

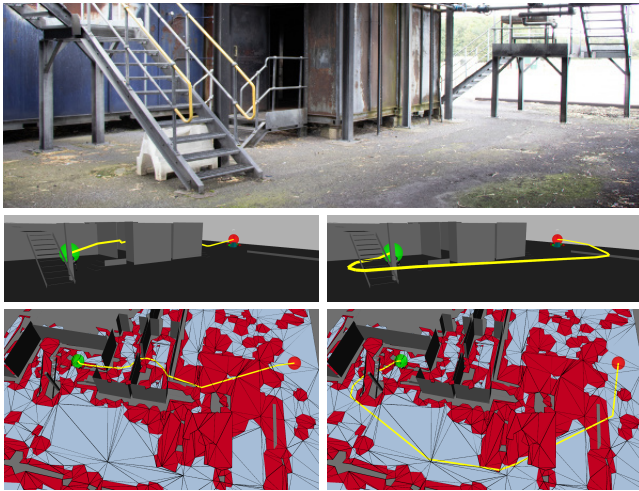


Fig. 6: The advantage of controller-reasoning in the NavMesh. Left: a least-distance path goes through buildings which require stair-climbing and few trotting. Right: the controller-aware path is of longer distance but 3.7 times faster to execute, since it goes around the buildings on long trotting periods.

one of the buildings. In addition, we added to the environment an object that was not present in the navigation mesh. Fig. 7 shows the robot trotting outside the building and adding virtual obstacles to the navigation mesh, indicated as red cylinders, as

it fails to locally plan to follow the global path. The robot then navigates around the new object and successfully climbs the stairs to access the inside of the building. Finally it walks to reach the desired goal. A video of the experiment is included in the attachment.

E. Long-range executions on the real robot

We conducted a set of field trials at the FSC oil rig (Fig. 1). In the first experiment the robot was placed on flat ground and given a goal around the facility, in a location that requires going inside and then navigating a set of containers connected through narrow passages. A zoom-up of the location is shown in Fig. 8, together with the trajectory executed by the robot (given by the localization system). The figure shows the robot trotting to the entrance, walking over the steps, navigating the containers, and exiting the structure through another set of steps on a distant side of the facility. The total traveled distance was 29 meters. Path-planning within the NavMesh took consistently 1ms throughout the execution.

For our second experiment the robot was placed outside on flat ground and given a goal straight ahead on the other side of the facility, after the 20cm barrier previously described in Sec. V-C. We set a goal close to the end of the barrier on purpose, so that a plan is produced that walks around the barrier instead of the shorter-distance option of climbing over—as in the experiment of Sec. V-C. Fig. 9 shows the path the robot takes, straight on flat ground, around the pillars and staircases and then around the barrier to reach the goal. The total traveled distance was 33 meters. Path-planning within the NavMesh took less than 1ms throughout.

Finally, we placed the robot in front of the same barrier but further away from its end, as in Fig. 5. The robot was given a goal straight ahead. Fig. 10 shows the robot trotting up to the barrier, walking over it using the vision-based walking controller and then trotting to the goal.

VI. CONCLUSION AND DISCUSSION

We proposed the use of navigation meshes as a high-level planning tool for long-range legged locomotion. We proposed a way to automatically annotate and build these structures in a way that is relevant to the multi-controller nature of legged robots. We integrated NavMeshes with high- and low-level planners that deal with long- and short-term reasoning, as

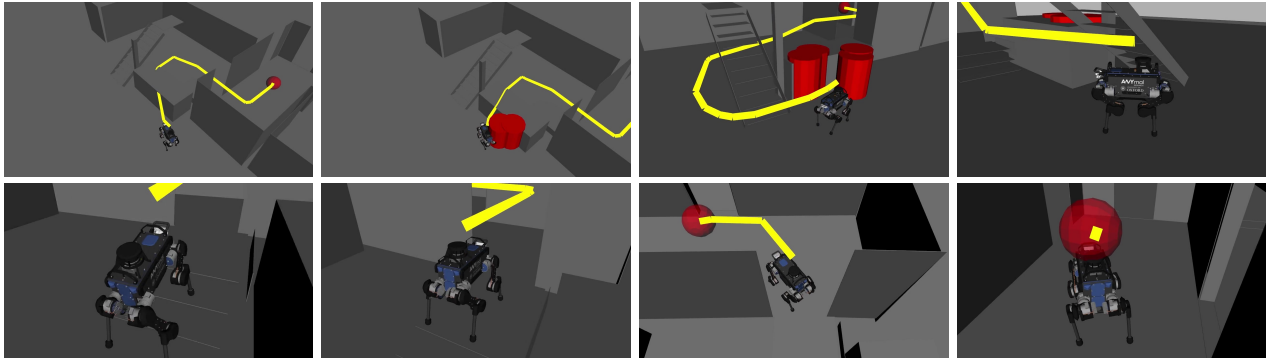


Fig. 7: Full execution in a simulated version of the FSC oil rig environment. Yellow lines indicate the shortest-path to the goal as computed in the NavMesh. The large box that the first paths go through was not modeled in the original NavMesh. Red cylinders indicate virtual obstacles added to the NavMesh on-line. The red sphere indicates the goal.

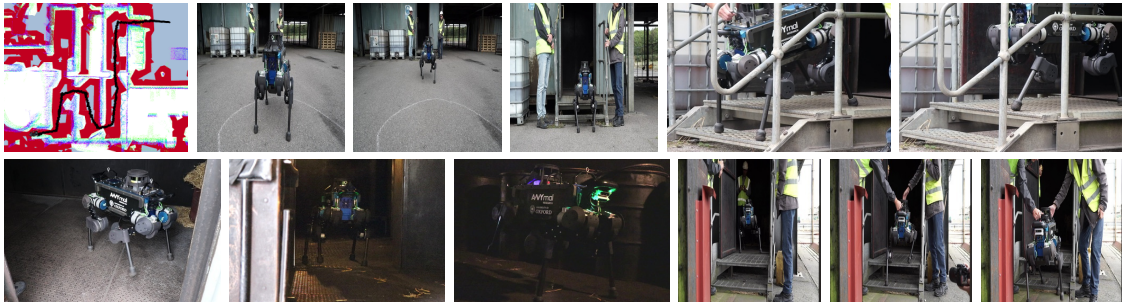


Fig. 8: Experiment 1 at the FSC oil rig site. The first image shows the original point cloud and navigation mesh (cropped for visibility) overlapped with the executed path given by localization.



Fig. 9: Experiment 2 at the FSC oil rig. The first image shows the original cloud, navigation mesh, and executed path given by localization. The robot trots through the facility and around a 20cm barrier to reach a goal close its end.

well as a way to switch between controllers and deal with unmodeled or new obstacles. We compared the performance of path-finding in NavMeshes against traditional sampling-based planners and quantitatively showed the superiority of NavMeshes—they are both faster to build and query than PRMs, as well as finding more paths. We concluded with a demonstration of the usefulness of such a system in real-world large-scale locomotion examples in an industrial facility.

In future work, we aim to tackle two of the limitations of the current system. One is the mesh reconstruction step, where because of point cloud noise can lead to narrow passages becoming even narrower on the reconstructed mesh and NavMesh. Currently this means that either robot-cylinder radii have to be made smaller than the actual robot for planning to be possible, or post-processing of the point cloud must be done to clean narrow passages before mesh reconstruction

(we used the former in this paper). In the future we will investigate better mesh reconstruction methods for this purpose. Another limitation of our approach is the cylindrical robot approximation in NavMeshes—which implies that NavMesh paths are not guaranteed to be executable in general and especially for long robots such as quadrupeds on narrow turns. To alleviate this issue we are considering a path-verification step of path planning or NavMesh-construction to identify such cases and obtain alternative paths. A limitation of the per-point assignment method is that it again uses statistics in a spherical region around the points, which can lead to conservative gait choices on narrow corridors if the sphere's size is large. Other interesting research directions include online NavMesh building, and the use of NavMeshes for sampling-bias or cost-heuristics in sampling and search-based multi-gait planning methods such as [1]. We also plan to improve the gait assignment method to

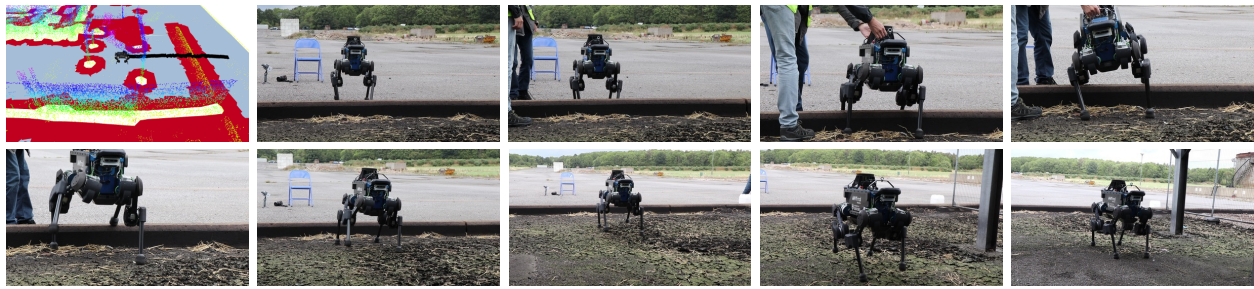


Fig. 10: Experiment 3 at the FSC oil rig. The first image shows the original cloud, navigation mesh, and executed path given by localization. The robot climbs over a 20cm barrier when far away from its end.

include the consideration of physical properties of the terrain such as friction, similarly to what is done in [32], [33].

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